Groups of Lie Type in MAGMA

Overview on some algorithms

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Outline

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Lie Theory in Magma

- Root systems and root data
- Coxeter groups

  (as permutation groups, as matrix reflection groups, as FP groups)
- Groups of Lie type (split and twisted)

  (Steinberg presentation, linear representations)
- Lie Algebras

  (as structure constants algebras, as matrix algebras)
- Universal Enveloping Algebras, Quantum Groups ...

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Groups of Lie Type

- Group of rational points of a linear algebraic group.
  (Subgroup of some $\mathrm{GL}_n(k)$ given by polynomials)
- Reductive groups are classified by root data.

Cartan type	Group
A_n	$\mathrm{SL}_{n+1}(k)$
B_n	$\mathrm{SO}_{2n+1}(k)$
C_n	$\mathrm{Sp}_{2n}(k)$
D_n	$\mathrm{SO}_{2n}(k)$
G_2, F_4, E_6, E_7, E_8	exceptional

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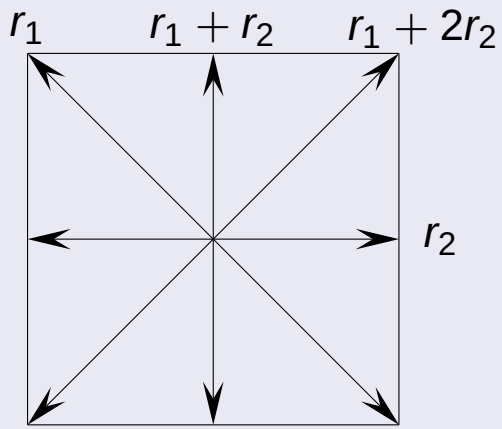
Root Systems

Root System of type B_2



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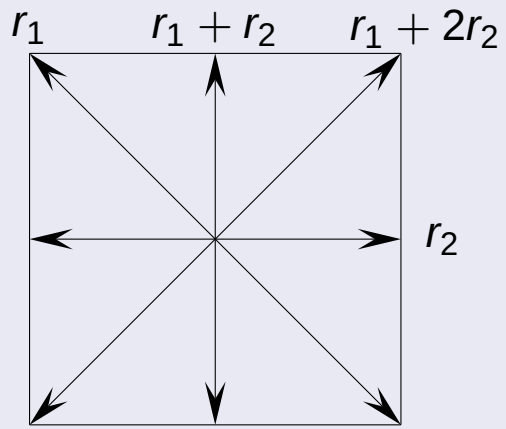

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- Π – Set of fundamental roots
- Φ^+ – Set of positive roots
- Φ^- – Set of negative roots

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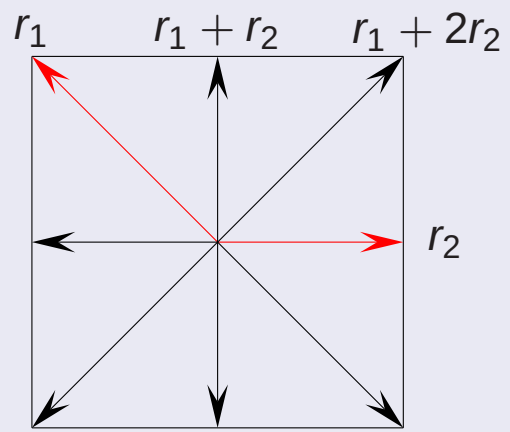

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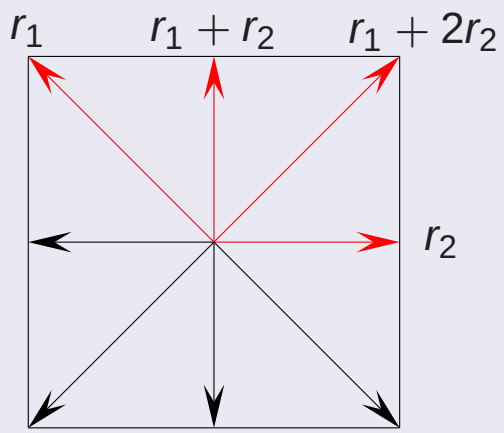

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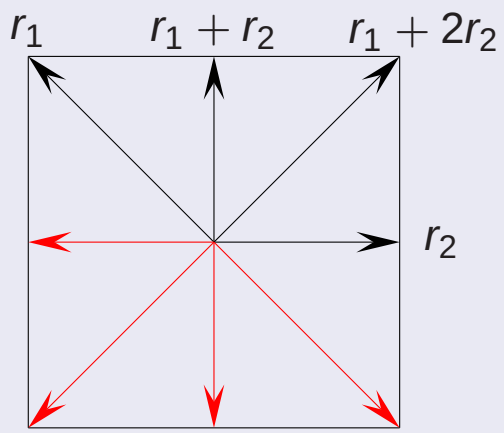

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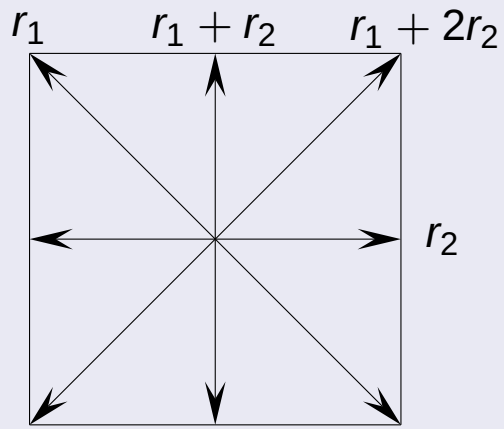

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Dynkin diagrams

Dynkin diagrams of reduced irreducible root systems.

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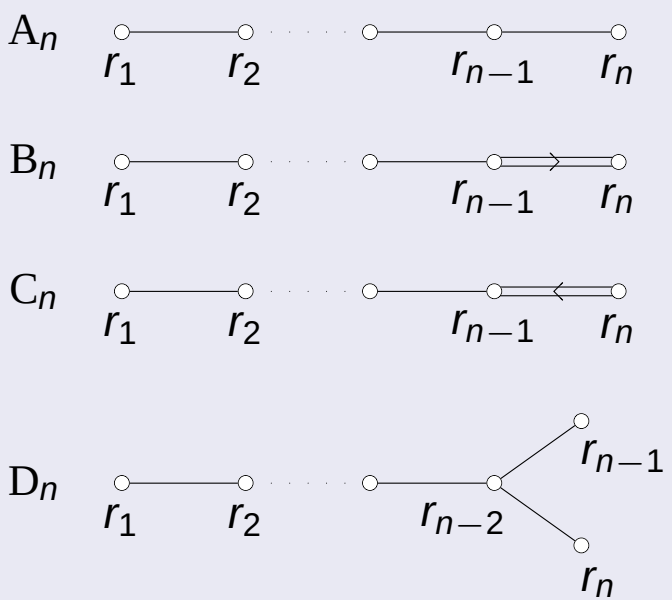

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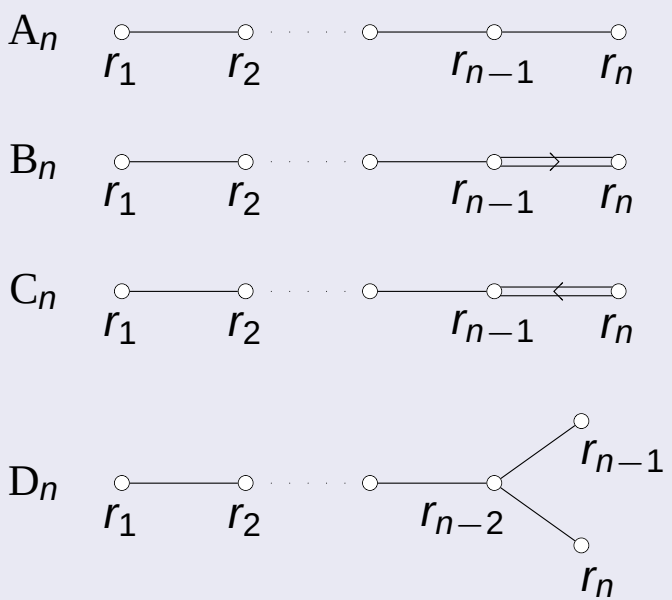

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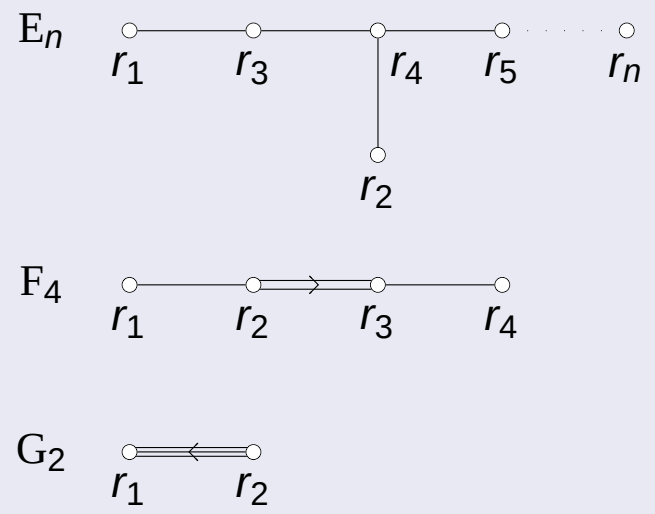

Steinberg Presentation

- Generators:
  - $x_r(t)$ with $r \in \Phi^+$ and $t \in k$ (unipotent)
  - $h_r(t)$ with $r \in \Pi$ and $t \in k^*$ (torus)
  - n_r with $r \in \Phi$ (Coxeter group elements)
- Relations:

$$x_r(t)x_r(u) = x_r(t+u)$$

$$[x_r(t), x_s(u)] = \prod_{i,j>0} x_{ir+js}(C_{rsij}t^i u^j)$$

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Unique Bruhat Decomposition

- $x = uhnu'$
  - u – product of $x_r(t)$
  - h – product of $h_r(t)$
  - n – product of n_r
  - u' – product of $x_r(t)$ [for a subset of Φ^+]
- Positive roots ordered with respect to their height. I.e., $r < s < r + s$ whenever $r < s$ and $r + s \in \Phi^+$.
- Unique decomposition of unipotent elements:

$$\prod_{r \in \Phi^+} x_r(t_r),$$

with roots in the above ordering.

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Question

How to compute?

Relations for Collection

$$x_r(0) = 1$$

$$x_r(t)x_r(u) = x_r(t+u)$$

$$x_r(t)x_s(u) = x_s(u)x_r(t) \qquad [\text{if } r+s \notin \Phi]$$

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Collection in General

Algorithm (Collection of u)

1. if u is in normal form, then stop
2. choose any non-normal subword of length 1 or 2 and collect it using above relations
3. go to step 1

- Always terminates! [Sims, 1987]
- Choice in step 2 is crucial for efficiency

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Collection To Left/From Right

Input: $\prod_{i=1}^{m} x_{r_i}(t_i)$ Output: $\prod_{r=1}^{N} x_r(t_r)$, where $N := |\Phi^+|$

Algorithm (Collection To Left) based on [Hall, 1934]
Algorithm (Collection From Right) [Neubüser, 1961]

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for r in [1..N] do
    collect all terms x_{r_i}(t_i) with r_i = r to left
    using the above relations
end for
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before step r: $\prod_{s=1}^{r-1} x_s(t_s) \cdot \prod_{i=1}^{m} x_{r_i}(t_i)$ $[r_i \geq r]$

after step r: $\prod_{s=1}^{r-1} x_s(t_s) \cdot x_r(\sum_{r_i=r} t_i) \cdot \prod_{i=1}^{m'} x_{r_i}(t_i)$ $[r_i > r]$

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Collection From Left

Input: $x = \prod_{i=1}^{m} x_{r_i}(t_i)$ Output: $x = \prod_{r=1}^{N} x_r(t_r)$

Algorithm (Collection From Left) [Leedham-Green, Soicher, 1989]

$k := \max\{1 \leq \ell \leq m \mid \prod_{i=1}^{\ell} x_{r_i}(t_i) \text{ is collected } \}$

`while` $k < m$ `do`

collect the term $x_{r_{k+1}}(t_{k+1})$ to left

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Rediscovered in [SH, 2000]

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Normal form
Using additive ordering on roots

- Positive roots ordered with respect to their height.
  I.e., $r < s < r + s$ whenever $r < s$ and $r + s \in \Phi^+$.
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  I.e., $r < r + s < s$ whenever $r < s$ and $r + s \in \Phi^+$.
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Algorithm (Collection From Outside) [Cohen,Haller,Murray, 2006]

- Basic Idea: Run at the same time
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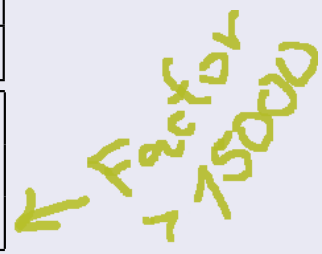

Abstract degrees of Hall Polynomials

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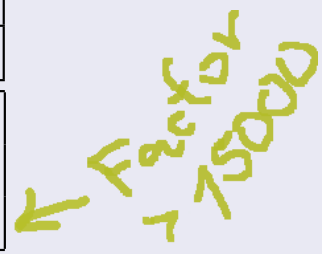
$B_{20}(17)$	290.450	1.290	0.146	2210.100



Abstract degrees of Hall Polynomials

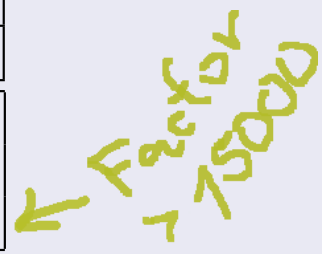
	CFL		CFO	
	max	avg	max	avg
A_n	n	$(n+2)/3$		
B_n	$2n-1$	$\sim (2n)/3$		
C_n	$2n-1$	$\sim (2n)/3$		
D_n	$2n-3$	$(2n-1)/3$		

Comparison of Algorithms

- AMD Opteron 150 (2393MHz)

Product of two random (collected) unipotent elements

	2.13			2.12
	CFR	CFL	CFO	CFL
$E_8(17)$	6.140	0.060	0.010	15.900
$A_{30}(17)$	128.600	0.797	0.085	155.170
$B_{20}(17)$	290.450	1.290	0.146	2210.100



Abstract degrees of Hall Polynomials

	CFL		CFO	
	max	avg	max	avg
A_n	n	$(n+2)/3$	2	$\to 2$
B_n	$2n-1$	$\sim (2n)/3$	4	$\to 4$
C_n	$2n-1$	$\sim (2n)/3$	3	$\to 3$
D_n	$2n-3$	$(2n-1)/3$	3	$\to 3$

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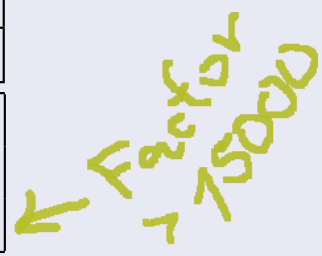

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In Type B_{15}, we have $|\Phi^+| = 225$ Hall Polynomials

Comparison of Algorithms

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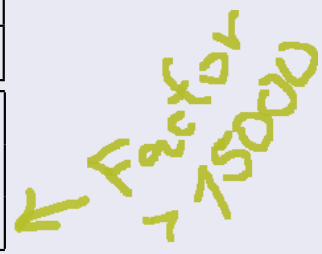

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	max	avg	max	avg
A_n	n	$(n+2)/3$	2	$\to 2$
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C_n	$2n-1$	$\sim (2n)/3$	3	$\to 3$
D_n	$2n-3$	$(2n-1)/3$	3	$\to 3$

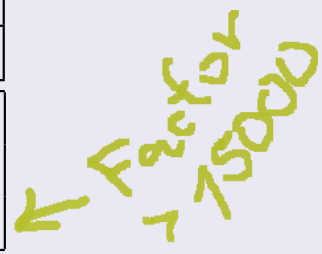
In Type B_{15}, we have $|\Phi^+| = 225$ Hall Polynomials
Largest (printed as string) is 9226 B with CFO

Comparison of Algorithms

- AMD Opteron 150 (2393MHz)

Product of two random (collected) unipotent elements

	2.13			2.12
	CFR	CFL	CFO	CFL
$E_8(17)$	6.140	0.060	0.010	15.900
$A_{30}(17)$	128.600	0.797	0.085	155.170
$B_{20}(17)$	290.450	1.290	0.146	2210.100



Abstract degrees of Hall Polynomials

	CFL		CFO	
	max	avg	max	avg
A_n	n	$(n+2)/3$	2	$\to 2$
B_n	$2n-1$	$\sim (2n)/3$	4	$\to 4$
C_n	$2n-1$	$\sim (2n)/3$	3	$\to 3$
D_n	$2n-3$	$(2n-1)/3$	3	$\to 3$

In Type B_{15}, we have $|\Phi^+| = 225$ Hall Polynomials
Largest (printed as string) is 9226 B with CFO and $\sim$ 297 MB with CFL.

Outline

Examples I

The following is a captured Magma session presented interactively at the conferecne. It demonstrates some of the new features available in Magma 2.13

```
Got mach: Intel(R) Pentium(R) M processor 2.00GHz
Magma V2.13-2     Tue Aug  1 2006 11:50:08 on 2go      [Seed = 15191]
Type ? for help.  Type <Ctrl>-D to quit.

Loading startup file "/home/sergei/.magmarc"

> iload demo;
Interactive-loading "demo"
>
> R := RootDatum("B2"); R;
R: Adjoint root datum of dimension 2 of type B2
> DynkinDiagram(R);

B2    1 =>= 2
> PositiveRoots(R);
{@
     (1 0),
     (0 1),
     (1 1),
     (1 2)
@}
>
```

Examples II

```
> R := RootDatum("BC2");
> DynkinDiagram(R);

BC2    1 =>= 2
> PositiveRoots(R);
{@
     (1 0),
     (0 1),
     (1 1),
     (0 2),
     (1 2),
     (2 2)
@}
>
> R := RootDatum( "A2" : Twist:=2 ); R;
R: Twisted adjoint root datum of dimension 2 of type 2A2,1
> RelativeRootDatum(R);
Adjoint root datum of dimension 1 of type BC1
> OrbitsPi(R);
[
     GSet{ 1, 2 }
]
> DistinguishedOrbitsPi(R);
[
     GSet{ 1, 2 }
]
>
```

Examples III

```
> R := RootDatum( "E6" : Twist:=<{{2}},2> ); R;
R: Twisted adjoint root datum of dimension 6 of type 2E6,1
> RelativeRootDatum(R);
Adjoint root datum of dimension 1 of type BC1
> OrbitsPi(R);
[
    GSet{ 2 },
    GSet{ 4 },
    GSet{ 1, 6 },
    GSet{ 3, 5 }
]
> DistinguishedOrbitsPi(R);
[
    GSet{ 2 }
]
>
> R := RootDatum( "D4" : Twist:=<{{2}},6> ); R;
R: Twisted adjoint root datum of dimension 4 of type 6D4,1
> RelativeRootDatum(R);
Adjoint root datum of dimension 1 of type BC1
>
```

Examples IV

```
> OrbitsPi(R);
[
    GSet{ 2 },
    GSet{ 1, 3, 4 }
]
> DistinguishedOrbitsPi(R);
[
    GSet{ 2 }
]
>
> R := RootDatum( "A3" : Twist:=2 ); R;
R: Twisted adjoint root datum of dimension 3 of type 2A3,2
> RelativeRootDatum(R);
Adjoint root datum of dimension 2 of type B2
> OrbitsPi(R);
[
    GSet{ 2 },
    GSet{ 1, 3 }
]
> DistinguishedOrbitsPi(R);
[
    GSet{ 2 },
    GSet{ 1, 3 }
]
>
```

Examples V

```
> R;
R: Twisted adjoint root datum of dimension 3 of type 2A3,2
> k := GF(5);
> K := GF(5^2);
> G := TwistedGroupOfLieType(R,k,K); G;
G: Twisted group of Lie type 2A3,2 over GF(5) with entries over GF(5^2)
>
> RelativeRootElement(G,1,[Random(K)]);
x2(2)
> RelativeRootElement(G,2,[Random(K)]);
x1($.1^21) x3($.1^9)
>
> quit;

Total time: 1.750 seconds, Total memory usage: 24.95MB
```

Outline

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