GALOIS COHOMOLOGY: SOME ASPECTS OF COMPUTATION AND APPLICATIONS

MAGMA WORKSHOP ON GROUP THEORY AND ALGEBRAIC GEOMETRY
UNIVERSITY OF WARWICK, COVENTRY, UK

SERGEI HALLER

ABSTRACT. This is a joint project with Arjeh M. Cohen, Scott H. Murray and D. E. Taylor. We design algorithms for computation with groups of Lie type and implement them in MAGMA [1]. The goal is to perform computations with parametrised group elements.

The untwisted groups of Lie type are groups of k-rational points of a connected reductive k-split linear algebraic group G defined over k. Algorithms for computing with elements of untwisted groups of Lie type are known and implemented (Cohen, Murray, Taylor [3]; Haller [4]; Riebeek [5]).

The twisted groups of Lie type are groups of rational points of twisted forms of G. The possible twists for a given field extension K over k are classified by Galois cohomology. We report on current work to make Galois cohomology effective. We will discuss a presentation for the untwisted groups of Lie type, which is similar to the Steinberg presentation for untwisted groups of Lie type.

Galois cohomology can also be used to compute all maximal tori of $G(k)$. For finite k, the tori are computed as subgroups $T_{\gamma w}$ of $G(K)_{\gamma w}$, where γ is the generator of $\mathrm{Gal}(K : k)$ and w is conjugation by an element normalising the split maximal torus T of $G(K)$. Using the effective Lang's theorem by Cohen and Murray [2], the tori can be conjugated from $G(K)_{\gamma w}$ back into the original group $G(k)$.

REFERENCES

[1] W. W. Bosma and J. J. Cannon, *Handbook of magma functions*, School of Mathematics and Statistics, University of Sydney, Sydney, 1997.

[2] Arjeh M. Cohen and Scott H. Murray, *Algorithm for Lang's theorem*, to appear.

[3] Arjeh M. Cohen, Scott H. Murray, and D. E. Taylor, *Computing in groups of Lie type*, Math. Comp. **73** (2004), 1477–1498. MR 2 047 097

[4] Sergei Haller, *Entwicklung und Implementierung eines Algorithmuses zur Berechnung von Kommutatoren unipotenter Elemente in Chevalley-Gruppen*, Diplomarbeit, Universität Gießen, 2000.

[5] R. J. Riebeek, *Computations in association schemes*, Ph.D. thesis, Eindhoven University of Technology, 1998.

Date: August 22–26, 2005.

Sergei.Haller@math.uni-giessen.de.